FRICTION DRAG OF TURBULENT FINELY DISPERSED AIR MIXTURES

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On the basis of a theoretical analysis of the turbulent stresses due to momentum transfer the author determines the excess drag coefficient in a finely dispersed air-mixture flow. The results obtained are checked against the experimental data.

In [1] it was shown that the dissipation of turbulent energy by the particles is insignificant as compared with the total drag of an air mixture. Since in finely dispersed air mixtures the bulk of the particles move in the suspended state and, in practice, the energy losses due to impact against the walls can be disregarded, we may assume that, as in the turbulent motion of pure air, the drag is associated with intense turbulent mixing. In this case as a result of the particles being transported together with the eddies there is an increase in turbulent stresses, which leads to an increase in friction at the flow boundaries.

In practical flow calculations wide use is made of the semiempirical theory of turbulence based on the ideas of momentum transfer in the flow cross section. Accordingly, it is of interest to apply the momentum transfer theory to the motion of an air mixture to obtain formulas for the drag.

In analyzing the particle transport processes the following assumptions will be made:

1) the principle of superposition of losses applies, i.e., the drags consist of the sum of the drags due to the motion of the pure air and the motion of the particles; the turbulent stresses are composed of the stresses due to the transporting of certain volumes of the medium and the additional stresses due to the transporting of the particles contained in those volumes;

2) the effect of the particles on the kinematic flow structure is insignificant;

3) the particles are quite uniformly distributed over the cross section;

4) the effect of gravity on the motion of the particles can be neglected;

5) the effect of particle impact against the walls is insignificant.

The turbulent stresses for pure air and an air mixture are respectively equal to

$$\theta_0 = -\rho_0 \overline{u_1' u_2'}, \tag{1}$$

$$\theta = -\left[\rho_0 (1-s)\overline{u_1' u_2'} + \rho_s s \overline{u_{s1}' u_{s2}'}\right]. \tag{2}$$

After transformations we obtain

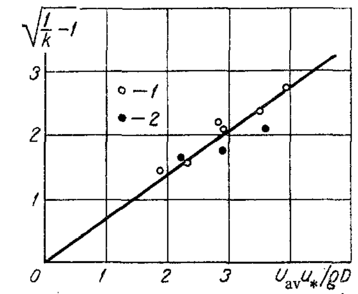
$$\theta = -\rho_0 (1-s)\overline{u_1' u_2'}\left[1 + \frac{\rho_s s}{\rho_0 (1-s)} \frac{\overline{u_{s1}' u_{s2}'}}{\overline{u_1' u_2'}}\right]. \tag{3}$$

In pneumatic transport the volume concentrations usually do not exceed one percent. Therefore we assume that $1 - s \approx 1$. Moreover, it is obvious that

$$\rho_s s/\rho_0 (1-s) = \mu.$$

Then from (3)

$$\theta = \theta_0 \left(1 + \mu \frac{\overline{u_{s1}' u_{s2}'}}{\overline{u_1' u_2'}}\right). \tag{4}$$



Dependence of $(1/k - 1)^{1/2}$ on the parameter $U_{av}u_*/gD$ according to the data of experiments on pneumatic transport of coal dust in a vertical tube with D = 0.054 m: 1) anthracite (ρ_s = 1810 kg/m^3, d_{av} = 0.0425 m, u_* = 0.077 m/sec); 2) bituminous coal (ρ_s = = 1660 kg/m^3, d_{av} = 0.044 mm, u_* = 0.078 m/sec).

The instantaneous value of the particle fluctuation velocity is given by [1, 2]

$$u_{si}' = \frac{u_{ai}'}{\sqrt{1+\omega_i^2\tau^2}} \sin(\omega_i t - \varphi). \tag{5}$$

Using (5), we obtain

$$\overline{u_{s1}' u_{s2}'} = \overline{\frac{u_{a1}'}{\sqrt{1+\omega_1^2\tau^2}} \sin(\omega_1 t - \varphi) \frac{u_{a2}'}{\sqrt{1+\omega_2^2\tau^2}} \sin(\omega_2 t - \varphi)}.$$

We assume that $\omega_1 \approx \omega_2 = \omega$. Then

$$\overline{u_{s1}' u_{s2}'} = \frac{1}{1+\omega^2\tau^2} \overline{u_{a1}' \sin(\omega t - \varphi) u_{a2}' \sin(\omega t - \varphi)}.$$

From a comparison of the right-hand side of the equation obtained and the approximate value of the fluctuation velocity of the medium $u_i' = u_{ai}' \sin \omega t$ [1] we see that under the averaging sign we have u_1' and u_2' shifted in phase. A phase shift of the instantaneous values does not affect the result of averaging over a large time interval. Consequently,

$$\overline{u_{s1}' u_{s2}'} = \frac{1}{1+\omega^2\tau^2} \overline{u_1' u_2'}$$

and (4) takes the form

$$\theta = \theta_0 \left(1 + \mu \frac{1}{1 + \omega^2\tau^2}\right). \quad (6)$$

The following formula is widely used for determining the drag:

$$i = i_0 (1 + k\mu), \quad (7)$$

where in the general case k depends on the physico-mechanical characteristics of the particles and the transport parameters.

Since the following relations [3] are valid for one-dimensional flow in a circular pipe with a pressure gradient:

$$\theta = \theta' \left(1 - \frac{2h}{D}\right) \quad (8)$$

and

$$\theta_0 = \theta_0' \left(1 - \frac{2h}{D}\right), \quad (9)$$

and the specific pressure losses are, respectively, equal to $i = 4\theta'/D$ and $i_0 = 4\theta_0'/D$, from (6)–(9) we find that

$$k = \frac{1}{1 + \omega^2\tau^2}. \quad (10)$$

The use of Eq. (10) is complicated by the fact that the particles are affected by fluctuations of the medium with a broad frequency spectrum. Since the effect of fluctuations of different frequencies on the motion of a particle is not the same, it is necessary to find the characteristic frequencies that play the decisive part in the motion of the particles and link them with the general flow parameters.

The general case of particle motion with allowance for the integral frequency spectrum was considered by Tchen in [4, 5]. For the case of particles moving in air, when $\rho_s \gg \rho_0$, from Tchen's results for $\omega \neq 0$ there follows

$$\frac{E_s}{E_0} = \frac{1}{1 + \omega^2\tau^2}. \quad (11)$$

From the relation obtained it follows that the excess drag coefficient is equal to the ratio of the energy spectra of the particles and the transport medium, which, generally speaking, is valid only if the frequencies of the longitudinal and transverse fluctuations are equal.

Since

$$\overline{(u')^2} = \int_0^\infty E_0(\omega)\, d\omega \text{ and } \overline{(u_s')^2} = \int_0^\infty E_s(\omega)\, d\omega,$$

from (11) for a narrow interval of frequencies ω_n we obtain

$$\overline{u'_{sn}} = \frac{\overline{u'_n}}{\sqrt{1 + \omega_n^2\tau^2}}. \quad (12)$$

(It should be noted that the same result can be obtained from Eq. (5), which corresponds to the special case of single-frequency motion).

The result obtained shows that the effect of fluctuations of different frequency on the particle fluctuation velocity decreases sharply as the frequency increases. In fact, $\overline{u'_n} = f(\omega_n)$, while for steady conditions (D = const, U_{av} = const) and fully developed turbulence we have [6]

$$\overline{u'} \sim (\varepsilon\delta)^{\frac{1}{3}}, \quad (13)$$

$$\omega \sim \overline{u'}/\delta = \varepsilon^{\frac{1}{3}}/\delta^{\frac{2}{3}}, \quad (14)$$

whence

$$\overline{u'_n} \sim (\varepsilon/\omega_n)^{\frac{1}{2}}, \quad (15)$$

where ε does not depend on the scale of turbulence and is a constant characteristic of the given flow.

$$\overline{u'_{sn}} \sim \left(\frac{\varepsilon}{\omega_n + \omega_n^3\tau^2}\right)^{\frac{1}{2}}. \quad (16)$$

Relation (16) gives the part of the total particle fluctuation velocity due to fluctuations in the narrow range of frequencies ω_n.

To estimate the effect of the different frequency components we used Laufer's experimental data on the turbulent flow spectrum in a circular pipe with $D = 0.25$ m at $U_{av} = 30$ m/sec [5]. The measured fluctuation frequencies ranged from $\omega_{min} = 90 - 150$ sec^{-1} to $\omega_{max} = 1.5 \cdot 10^5$ sec^{-1}. Calculations based on (16) for coal particles of density 1660 kg/m^3 with $\tau = 10^{-2}$ sec ($d = 0.044$ mm) show that on transition from frequencies $\omega_{min} \approx 10^2$ sec^{-1} to frequencies $\omega_n = 10^3$ sec^{-1}, i.e., in a region comprising roughly 0.01 of the entire frequency range, the quantity $\overline{u'_{sn}}$ decreases by a factor of 20. From this it follows that the particle fluctuation velocity is chiefly determined by the fluctuations of the medium on only a narrow interval of minimum frequencies $\omega \approx \omega_{min}$, where in accordance with (14)

$$\omega_{min} \sim U_{av}/D, \quad (17)$$

and the proportionality factor is on the order of unity [6]. For the conditions of Laufer's experiments with a proportionality factor equal to unity from (17) we obtain $\omega_{min} = 120$ sec^{-1}, which is in good agreement with the data presented above.

It should be noted that the experimental data indicate the approximate equality of the minimum frequencies for the longitudinal and transverse fluctuation components [5], which confirms the validity of our assumption that $\omega_1 \approx \omega_2$.

Assuming that the decisive influence is exerted by the minimum frequency fluctuations and considering that $\tau = u_*/g$ [1], from (10) and (17) we obtain

$$k = \frac{1}{1 + \left(c\, \frac{U_{av} u_*}{gD}\right)^2}, \quad (18)$$

where the proportionality factor c is found from an analysis of the experimental data and may be assumed to be on the order of unity.

The figure shows the relation $(1/k - 1)^{1/2} = f(U_{av}u_*/gD)$ based on the data of experiments on the pneumatic transport of coal dust in a vertical tube with D = 0.054 m [7]. It is clear from the graph that the experimental data are in rather good agreement with the linear relation corresponding to c = 0.7. Equation (18) can be recommended for use in connection with the pneumatic transport of materials in dust form; however, because of the limited amount of available experimental data the value obtained for c requires further verification.

In calculating the drags for ascending flows we should take into account the energy losses associated with lifting the particles. In the experimental data presented [7] only the friction drag without the weight of the column of air mixture is represented.

It follows from (18) that, as the flow velocity increases, the excess drag coefficient decreases, although the total drag increases owing to the increase in i_0. Moreover, as $u_* \to 0$, $k \to 1$ and as $u_* \to \infty$, $k \to 0$. This is because in the case of very small particles, which are almost completely entrained by the turbulent fluctuations, the motion of the air mixture is similar to that of a homogeneous fluid of increased density $\rho = \rho_0(1+\mu)$. On the other hand, large particles are hardly entrained at all by the fluctuations of the medium, as a result of which there is no turbulent momentum transfer; in this case particle transport and the corresponding drag depend on another mechanism associated with the action of the averaged flow velocity gradient on the particles.

NOTATION

u_i' and u_{si}' (i is the coordinate index) denote the fluctuation velocity of the air and the particles, respectively; $\overline{u'}$ and $\overline{u_s'}$ denote the mean-square fluctuation velocity of the air and the particles, respectively; u_a' is the amplitude of the air fluctuation velocity; U_{av} is the average flow velocity; u_* is the critical particle velocity; ω is the angular frequency of the fluctuations; ρ_0, ρ_s, and ρ denote the density of air, particles, and air mixture, respectively; τ is the particle relaxation time; D is the pipe diameter; h is the distance from the pipe wall; d is the particle diameter; δ is the scale of turbulence; ε is the energy dissipated per unit time per unit mass of medium; θ_0 and θ are the turbulent stresses in a flow of pure air and air mixture; respectively; θ_0' and θ' denote stress at pipe wall for pure air and air mixture; t is the time; φ is the phase angle; s is the averaged volume concentration at point in flow; μ is the mass concentration of the air mixture; i_0 and i are the relative drags for pure air and air mixture, respectively; E_s and E_0 are the functions of energy spectrum for particles and medium; k is the experimental excess drag coefficient associated with the presence of particles in the flow; c is the proportionality factor.

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